Model of the synchronous machine in RAMSES

1 Flux-current relationships

In order to have a single model, whatever the number of rotor windings, we use "model switches", i.e. integer parameters defined as follows:

S_{d1} = 1 if there is a damper winding $d1$, = 0 otherwise

S_{q1} = 1 if there is a damper winding $q1$, = 0 otherwise

S_{q2} = 1 if there is a equivalent winding $q2$, = 0 otherwise

The table below shows usual models and their corresponding values of the model switches.

model	switches
detailed, round rotor	$S_{d1} = 1$, $S_{q1} = 1$, $S_{q2} = 1$
detailed, salient pole	$S_{d1} = 1$, $S_{q1} = 1$, $S_{q2} = 0$
simplified, no damper	$S_{d1} = 0$, $S_{q1} = 0$, $S_{q2} = 0$

Using the Equal-Mutual-Flux-Linkage (EMFL) per unit system, the relationship between magnetic flux linkages and currents can be written as:

$$\begin{bmatrix} \psi_d \\ \psi_f \\ \psi_{d1} \end{bmatrix} = \begin{bmatrix} L_\ell + M_d & M_d & S_{d1}M_d \\ M_d & L_{\ell f} + M_d & S_{d1}M_d \\ S_{d1}M_d & S_{d1}M_d & L_{\ell d1} + S_{d1}M_d \end{bmatrix} \begin{bmatrix} i_d \\ i_f \\ i_{d1} \end{bmatrix}$$

$$\begin{bmatrix} \psi_q \\ \psi_{q1} \\ \psi_{q2} \end{bmatrix} = \begin{bmatrix} L_\ell + M_q & S_{q1}M_q & S_{q2}M_q \\ S_{q1}M_q & L_{\ell q1} + S_{q1}M_q & S_{q2}M_q \\ S_{q2}M_q & S_{q2}M_q & L_{\ell q2} + S_{q2}M_q \end{bmatrix} \begin{bmatrix} i_q \\ i_{q1} \\ i_{q2} \end{bmatrix}$$

The d and q components of the air-gap flux are given by:

$$\psi_{ad} = M_d(i_d + i_f + S_{d1}i_{d1}) \quad (1)$$

$$\psi_{aq} = M_q(i_q + S_{q1}i_{q1} + S_{q2}i_{q2}) \quad (2)$$

From the above equations, and taking into account that the S switches can take values in $\{0, 1\}$ only, one easily obtains:

$$\begin{aligned}
\psi_d &= L_\ell i_d + \psi_{ad} && (3)\\
\psi_f &= L_{\ell f} i_f + \psi_{ad} && (4)\\
\psi_{d1} &= L_{\ell d1} i_{d1} + S_{d1}\psi_{ad} && (5)\\
\psi_q &= L_\ell i_q + \psi_{aq} && (6)\\
\psi_{q1} &= L_{\ell q1} i_{q1} + S_{q1}\psi_{aq} && (7)\\
\psi_{q2} &= L_{\ell q2} i_{q2} + S_{q2}\psi_{aq} && (8)
\end{aligned}$$

Using (4, 5, 7, 8), the rotor currents are obtained from flux linkages as:

$$\begin{aligned}
i_f &= \frac{\psi_f - \psi_{ad}}{L_{\ell f}} && (9)\\
i_{d1} &= \frac{\psi_{d1} - S_{d1}\psi_{ad}}{L_{\ell d1}} && (10)\\
i_{q1} &= \frac{\psi_{q1} - S_{q1}\psi_{aq}}{L_{\ell q1}} && (11)\\
i_{q2} &= \frac{\psi_{q2} - S_{q2}\psi_{aq}}{L_{\ell q2}} && (12)
\end{aligned}$$

2 Saturation model

Let M_d^u and M_q^u be the *unsaturated* direct- and quadrature-axis mutual inductances, related to their corresponding saturated values M_d and M_q by:

$$\begin{aligned}
M_d &= \frac{M_d^u}{1 + m\left(\sqrt{\psi_{ad}^2 + \psi_{aq}^2}\right)^n} && (13)\\
M_q &= \frac{M_q^u}{1 + m\left(\sqrt{\psi_{ad}^2 + \psi_{aq}^2}\right)^n} && (14)
\end{aligned}$$

Replacing M_d by the above expression and i_f and i_{d1} by (9, 10), respectively, the expression (1) of the d-axis air-gap flux becomes:

$$\psi_{ad} = \frac{M_d^u}{1 + m\left(\sqrt{\psi_{ad}^2 + \psi_{aq}^2}\right)^n}\left(i_d + \frac{\psi_f - \psi_{ad}}{L_{\ell f}} + S_{d1}\frac{\psi_{d1} - S_{d1}\psi_{ad}}{L_{\ell d1}}\right)$$

Rearranging terms yields the algebraic equation[1]:

$$\psi_{ad}\left(\frac{1 + m\left(\sqrt{\psi_{ad}^2 + \psi_{aq}^2}\right)^n}{M_d^u} + \frac{1}{L_{\ell f}} + \frac{S_{d1}}{L_{\ell d1}}\right) - i_d - \frac{1}{L_{\ell f}}\psi_f - \frac{S_{d1}}{L_{\ell d1}}\psi_{d1} = 0 \qquad (15)$$

[1] noting that $S_{d1}^2 = S_{d1}$

Similarly we obtain for the q axis:

$$\psi_{aq}\left(\frac{1+m\left(\sqrt{\psi_{ad}^2+\psi_{aq}^2}\right)^n}{M_q^u}+\frac{S_{q1}}{L_{\ell q1}}+\frac{S_{q2}}{L_{\ell q2}}\right)-i_q-\frac{S_{q1}}{L_{\ell q1}}\psi_{q1}-\frac{S_{q2}}{L_{\ell q2}}\psi_{q2}=0 \tag{16}$$

3 Reference frame

The d and q components of the stator voltage relate to their x and y components through:

$$\begin{pmatrix} v_d \\ v_q \end{pmatrix} = \begin{pmatrix} -\sin\delta & \cos\delta \\ \cos\delta & \sin\delta \end{pmatrix} \begin{pmatrix} v_x \\ v_y \end{pmatrix} \tag{17}$$

and similarly for the current:

$$\begin{pmatrix} i_d \\ i_q \end{pmatrix} = \begin{pmatrix} -\sin\delta & \cos\delta \\ \cos\delta & \sin\delta \end{pmatrix} \begin{pmatrix} i_x \\ i_y \end{pmatrix} \tag{18}$$

Eqs. (15, 16) become respectively:

$$\psi_{ad}\left(\frac{1+m(\sqrt{\psi_{ad}^2+\psi_{aq}^2})^n}{M_d^u}+\frac{1}{L_{\ell f}}+\frac{S_{d1}}{L_{\ell d1}}\right)+\sin\delta\, i_x-\cos\delta\, i_y-\frac{1}{L_{\ell f}}\psi_f-\frac{S_{d1}}{L_{\ell d1}}\psi_{d1}=0 \tag{19}$$

Similarly we obtain for the q axis:

$$\psi_{aq}\left(\frac{1+m(\sqrt{\psi_{ad}^2+\psi_{aq}^2})^n}{M_q^u}+\frac{S_{q1}}{L_{\ell q1}}+\frac{S_{q2}}{L_{\ell q2}}\right)-\cos\delta\, i_x-\sin\delta\, i_y-\frac{S_{q1}}{L_{\ell q1}}\psi_{q1}-\frac{S_{q2}}{L_{\ell q2}}\psi_{q2}=0 \tag{20}$$

4 Park equations

The original Park equations are written as:

$$v_d = -R_a i_d - \omega\psi_q \tag{21}$$

$$v_q = -R_a i_q + \omega\psi_d \tag{22}$$

$$\frac{d\psi_f}{dt} = \omega_N(K_f v_f - R_f i_f) \tag{23}$$

$$\frac{d\psi_{d1}}{dt} = -\omega_N R_{d1} i_{d1} \tag{24}$$

$$\frac{d\psi_{q1}}{dt} = -\omega_N R_{q1} i_{q1} \tag{25}$$

$$\frac{d\psi_{q2}}{dt} = -\omega_N R_{q2} i_{q2} \tag{26}$$

The stator equations are transformed as follows. Eqs. (18) are used to express v_d, v_q, i_d and i_q as functions of v_x, v_y, i_x, i_y, while Eqs. (3, 6) are used to involve ψ_{ad} and $\psi_a q$. This yields for the d axis:

$$\begin{aligned} v_d &= -R_a(-\sin\delta\, i_x + \cos\delta\, i_y) - \omega(L_\ell i_q + \psi_{aq}) \\ &= -R_a(-\sin\delta\, i_x + \cos\delta\, i_y) - \omega L_\ell(\cos\delta\, i_x + \sin\delta\, i_y) - \omega\psi_{aq} \end{aligned}$$

and finally:

$$0 = \sin\delta v_x - \cos\delta v_y + (R_a \sin\delta - \omega L_\ell \cos\delta) i_x - (R_a \cos\delta + \omega L_\ell \sin\delta) i_y - \omega\psi_{aq} \tag{27}$$

For the q axis we have:

$$\begin{aligned} v_q &= -R_a(\cos\delta\, i_x + \sin\delta\, i_y) + \omega(L_\ell i_d + \psi_{ad}) \\ &= -R_a(\cos\delta\, i_x + \sin\delta\, i_y) + \omega L_\ell(-\sin\delta\, i_x + \cos\delta\, i_y) + \omega\psi_{ad} \end{aligned} \tag{28}$$

and finally:

$$0 = -\cos\delta v_x - \sin\delta v_y - (R_a \cos\delta + \omega_N L_\ell \sin\delta) i_x - (R_a \sin\delta - \omega L_\ell \cos\delta) i_y + \omega\psi_{ad} \tag{29}$$

The rotor equations are transformed by substituting the expressions (9, 10, 11, 12) for the rotor currents, thereby obtaining:

$$\frac{d\psi_f}{dt} = \omega_N(K_f v_f - R_f \frac{\psi_f - \psi_{ad}}{L_{\ell f}}) \tag{30}$$

$$\frac{d\psi_{d1}}{dt} = -\omega_N R_{d1} \frac{\psi_{d1} - S_{d1}\psi_{ad}}{L_{\ell d1}} \tag{31}$$

$$\frac{d\psi_{q1}}{dt} = -\omega_N R_{q1} \frac{\psi_{q1} - S_{q1}\psi_{aq}}{L_{\ell q1}} \tag{32}$$

$$\frac{d\psi_{q2}}{dt} = -\omega_N R_{q2} \frac{\psi_{q2} - S_{q2}\psi_{aq}}{L_{\ell q2}} \tag{33}$$

5 Rotor motion

The equations of the rotor motion are:

$$\frac{1}{\omega_N}\frac{d\delta}{dt} = \omega - \omega_{coi} \tag{34}$$

$$2H\frac{d\omega}{dt} = K_m T_m - T_e - D(\omega - \omega_{coi}) \tag{35}$$

in which the expression of the electromagnetic torque T_e is obtained as follows:

$$\begin{aligned} T_e &= \psi_d i_q - \psi_q i_d = (L_\ell i_d + \psi_{ad}) i_q - (L_\ell i_q + \psi_{aq}) i_d \\ &= \psi_{ad} i_q - \psi_{aq} i_d = \psi_{ad}(\cos\delta\, i_x + \sin\delta\, i_y) - \psi_{aq}(-\sin\delta\, i_x + \cos\delta\, i_y) \end{aligned} \tag{36}$$

6 Unknowns-equations balance

The 10 state variables are: i_x, i_y, ψ_{ad}, ψ_{aq}, ψ_f, ψ_{d1}, ψ_{q1}, ψ_{q2}, δ, ω.

They are balanced by:

- 4 algebraic equations: (19, 20, 27, 29)
- 6 differential equations: (30, 31, 32, 33, 34, 35)

7 On the per unit system

The above model relies on the EMFL per unit system, which is the most convenient for a detailed model of the synchronous machine such as the one above. On the other hand, it is quite common to have the excitation system modelled in its own per unit system. A change of base current/voltage is thus necessary to interface both models.

While the EMFL system has been chosen for the synchronous machine, the user may use his/her own per unit system for the excitation system. The latter has to be specified through the parameter IBRATIO defined hereafter.

The base current of the field winding is I_{fB}^{mac} in the machine model and I_{fB}^{exc} in the excitation system model. The ratio of these two bases is defined as :

$$IBRATIO = \frac{I_{fB}^{mac}}{I_{fB}^{exc}}. \tag{37}$$

A given field current i_f (in A) has the following value in per unit on the machine base :

$$i_{f,pu}^{mac} = \frac{i_f}{I_{fB}^{mac}} \tag{38}$$

and the following value in per unit on the excitation system base :

$$i_{f,pu}^{exc} = \frac{i_f}{I_{fB}^{exc}} \tag{39}$$

By combining Eqs. (37, 38, 39), it is easily found that :

$$IBRATIO = \frac{i_{f,pu}^{exc}}{i_{f,pu}^{mac}}. \tag{40}$$

Three examples follow.

7.1 Open-circuit unsaturated machine

In this per unit system, which is most often used, I_{fB}^{exc} is the field current which produces the nominal stator voltage ($V = 1$ pu) when the machine rotates at its nominal speed ($\omega = 1$ pu) with its stator open ($i_d = i_q = 0$), saturation being neglected.

In these operating conditions, the machine Park equations give :

$$v_d = \psi_q = \psi_{aq} = 0 \qquad v_q = \psi_d = \psi_{ad} = M_d^u \, i_{f,pu}^{mac}$$

and, hence :

$$V = \sqrt{v_d^2 + v_q^2} = 1 \quad \Rightarrow \quad M_d^u \, i_{f,pu}^{mac} = 1$$

On the excitation system base :

$$i_{f,pu}^{exc} = 1$$

Introducing the last two relations in (40) yields :

$$IBRATIO = M_d^u = X_d^u - X_\ell$$

7.2 Open-circuit saturated machine

In this per unit system, I_{fB}^{exc} is the field current which produces the nominal stator voltage ($V = 1$ pu) when the machine rotates at its nominal speed ($\omega = 1$ pu) with its stator open ($i_d = i_q = 0$), saturation being taken into account.

In these conditions, the machine Park equations give :

$$v_d = \psi_q = \psi_{aq} = 0 \qquad v_q = \psi_d = \psi_{ad} = M_d \, i_{f,pu}^{mac}$$

and, hence :

$$V = \sqrt{v_d^2 + v_q^2} = 1 \quad \Rightarrow \quad M_d \, i_{f,pu}^{mac} = 1$$

The saturation model (13) gives :

$$M_d = \frac{M_d^u}{1 + m\,\psi_{ad}^n} = \frac{M_d^u}{1 + m\,(M_d \, i_{f,pu}^{mac})^n} = \frac{M_d^u}{1 + m}$$

On the excitation system base :

$$i_{f,pu}^{exc} = 1$$

Introducing the last three relations in (40) yields :

$$IBRATIO = \frac{M_d^u}{1 + m} = \frac{X_d^u - X_\ell}{1 + m}$$

7.3 Saturated machine at nominal operating conditions

In this per unit system, I_{fB}^{exc} is the field current when the machine, rotating at its nominal speed ($\omega = 1$ pu), produces its nominal active and reactive powers under its nominal stator voltage ($V = 1$ pu, $P = \cos\phi_N$ and $Q = \sin\phi_N$), saturation being taken into account.